**Math 60 7.3 Adding and Subtracting Rational Expressions with Common Denominator**

Objectives:

1) Add two rational expressions
   a. Add numerators by combining like terms
   b. Keep common denominator unchanged
   c. AFTER adding, factor completely and simplify if possible
2) Subtract two rational expressions
   a. Subtract numerators by distributing negative to all terms of second numerator
   b. Keep common denominator unchanged
   c. AFTER subtracting, factor completely and simplify if possible
3) Add or subtract two rational expressions after factoring out -1 from one denominator
   a. Recognize $(b-a)=-(a-b)$ and substitute
   b. Move negative to numerator
   c. Proceed as before.

**Practice:**

Factor out -1 and rewrite the terms in the opposite order.

1) $3-t$ 2) $y^2-x^2$ 3) $49-x^2$ 4) $100-81x^2$

Perform the indicated operations and simplify.

5) $\dfrac{2x^2+x}{x^2-4}+\dfrac{x-x^2}{x^2-4}$

6) $\dfrac{2n^2}{n^2-1}-\dfrac{2n}{n^2-1}$

7) $\dfrac{2x^2-5x}{3x}+\dfrac{x^2-13x}{3x}$

8) $\dfrac{w^2-3w}{w-4}+\dfrac{4}{4-w}$

9) $\dfrac{b^2-11}{b^2-25}-\dfrac{3b+1}{25-b^2}$

10) $\dfrac{2k}{6k-6}-\dfrac{9+4k}{6-6k}$

11) $\dfrac{2p-3q}{3p^2-18q^2}+\dfrac{9q+5p}{18q^2-3p^2}$

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Warm-up.

(1) Add:

$$\frac{1}{6} + \frac{7}{6}$$

← denominators are the same. This is called having a common denominator (or CD)

$$= \frac{1+7}{6}$$

← add numerators
← copy CD unchanged

$$= \frac{8}{6}$$

LAST: reduce (simplify)

$$= \boxed{\frac{4}{3}}$$

(2) Add

$$\frac{1}{6} + \frac{7}{-6}$$

← $6 \neq -6$ so we do not have a common denominator. But it's close!

$$\frac{7}{-6} = -\frac{7}{6} = \frac{-7}{6}$$

↑ 3 locations for the negative are equivalent; negative in denominator is not simplified.

$$= \frac{1}{6} + \left(-\frac{7}{6}\right)$$

← move negative to an equivalent location. (In front works best.) NOW we have a common denominator.

$$= \frac{1}{6} - \frac{7}{6}$$

Proceed as before.

$$= \frac{1-7}{6}$$

$$= -\frac{6}{6} = \boxed{-1}$$

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Our goal: (8) $\frac{w^2-3w}{w-4} + \frac{4}{4-w}$

Notice: Denominators are NOT the same

$w-4$

$4-w$

example: if $w=3$: $w-4 \Rightarrow 3-4=-1$

$4-w \Rightarrow 4-3=+1$

IMPORTANT: We cannot add or subtract rational expressions until after we have a common denominator.

Factor out GCF $-1$ and rewrite:

(1) $3-t$

$= -1(-3+t)$

$= \boxed{-(t-3)}$

(2) $y^2-x^2$

$= -1(-y^2+x^2)$

$= \boxed{-(x^2-y^2)}$

(3) $49-x^2$

$= -1(-49+x^2)$

$= \boxed{-(x^2-49)}$

(4) $100-81x^2$

$= -1(-100+81x^2)$

$= \boxed{-(81x^2-100)}$

Note: All of these problems can be done by shortcut.

Notice that:

- you can change the order of subtract IF
- you put a $-1$ GCF in front.

Note: This is only for subtraction.

Remember the three locations for a negative

$$-\frac{1}{3} = \frac{-1}{3} = \frac{1}{-3}$$

We will need to move a negative out of the denominator

Math 60 7.3 Adding and Subtracting Rational Expressions that already have a common denominator

Solutions to Practice:

(5) $\frac{2x^2+x}{x^2-4} + \frac{x-x^2}{x^2-4}$ step 1: notice common denominator

$= \frac{2x^2+x+x-x^2}{x^2-4}$ step 2: add numerators, keep denominator

$= \frac{x^2+2x}{x^2-4}$ combine like terms in numerator.

$= \frac{x(x+2)}{(x+2)(x-2)}$ step 3: Factor completely & divide out common factors. (Simplify)

$= \boxed{\frac{x}{x-2}}$

(6) $\frac{2n^2}{n^2-1} - \frac{2n}{n^2-1}$ step 1: Notice common denominator

$= \frac{2n^2-2n}{n^2-1}$ step 2: Subtract numerators, keep denom.

$= \frac{2n(n-1)}{(n+1)(n-1)}$ step 3: Simplify by factor & cancel.

$= \boxed{\frac{2n}{n+1}}$

(7) $\frac{2x^2-5x}{3x} - \frac{x^2-13x}{3x}$ step 1 CD.

$= \frac{2x^2-5x-(x^2-13x)}{3x}$

$= \frac{2x^2-5x-x^2+13x}{3x}$ step 2: subtract by dist negative

$= \frac{x^2+8x}{3x}$ Combine like terms

$= \frac{x(x+8)}{3\cdot x} = \boxed{\frac{x+8}{3}}$ step 3 Divide out $\frac{x}{x}=1$.

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Perform indicated operations and simplify completely.

(8) $\dfrac{w^2-3w}{w-4} + \dfrac{4}{4-w}$

↑ We will factor out $-1$ from this denominator

$= \dfrac{w^2-3w}{w-4} + \dfrac{4}{-(w-4)}$

↑ We still do not have a common denominator because of this negative.

We can move the negative to
a) the front of this fraction * usually works better.
or b) the numerator of this fraction.

$= \dfrac{w^2-3w}{w-4} - \dfrac{4}{w-4}$ OR $\dfrac{w^2-3w}{w-4} + \dfrac{-4}{w-4}$

↑ front ↗ numerator

NOW WE HAVE A COMMON DENOMINATOR!

$= \dfrac{w^2-3w-4}{w-4}$

$= \dfrac{(w-4)(w+1)}{(w-4)}$

$-4$, $+1$, $-3$

$= \boxed{w+1}$

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(9) $\frac{b^2-11}{b^2-25} - \frac{3b+1}{25-b^2}$

step 1: Notice that we do NOT have a common denominator. Factor out $-1$, change order of subtraction.

$= \frac{b^2-11}{b^2-25} - \frac{3b+1}{-1(b^2-25)}$

step 2: Move negative to front or numerator use ( ) to dist to both terms of numerator.

$= \frac{b^2-11}{b^2-25} - \frac{-(3b+1)}{b^2-25}$

$= \frac{b^2-11}{b^2-25} + \frac{3b+1}{b^2-25}$

Mult neg by neg in this problem.

$= \frac{b^2-11+3b+1}{b^2-25}$

step 3: Add numerators

$= \frac{b^2+3b-10}{b^2-25}$

combine like terms

$= \frac{(b+5)(b-2)}{(b-5)(b+5)}$

factor completely

$= \boxed{\frac{b-2}{b-5}}$

divide out common factors.

(10) $\frac{2k}{6k-6} - \frac{9+4k}{6-6k}$

not a CD.

$= \frac{2k}{6k-6} - \frac{9+4k}{-(6k-6)}$

factor out $-1$, change order of subtraction.

$= \frac{2k}{6k-6} + \frac{9+4k}{6k-6}$

move $-1$ to front of fraction.

$= \frac{2k+9+4k}{6k-6}$

add numerators

$= \frac{6k+9}{6k-6}$

combine like terms

$= \frac{3(2k+3)}{6(k-1)} = \boxed{\frac{(2k+3)}{2(k-1)}}$

factor & cancel.

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(11) $$\frac{2p-3q}{3p^2-18q^2} + \frac{9q+5p}{18q^2-3p^2}$$

← We do not have a common denominator.

$$= \frac{2p+3q}{3p^2-18q^2} + \frac{9q+5p}{-(3p^2-18q^2)}$$

← factor out $-1$ from second denominator

$$= \frac{2p+3q}{3p^2-18q^2} - \frac{9q+5p}{3p^2-18q^2}$$

← move this $-1$ to the front of fraction, changing add to subtract

$$= \frac{(2p+3q)-(9q+5p)}{3p^2-18q^2}$$

← subtract numerators.

*VERY IMPORTANT*

We must subtract the entire second numerator, which means

1. use ( )
2. distribute negative

$$= \frac{2p+3q-9q-5p}{3p^2-18q^2}$$

Caution: The variables p and q are in a different order in second fraction.

Combine like terms carefully

$2p-5p$

and $3q-9q$

(NOT: $2p-9q$ or $3q-5p$)

$$= \frac{-3p-6q}{3p^2-18q^2}$$

$$= \frac{-3(p+2q)}{3(p^2-6q^2)}$$

$$= \boxed{\frac{-(p+2q)}{(p^2-6q^2)}}$$